## LINEAR INTEGRATED CIRCUITS

## PART-04 <br> AC Analysis of BJT Differential Amplifier Circuit

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## AC Analysis

AC analysis of a BJT Differential Amplifier involves determination of

1. Voltage Gain
2. Input Impedance
3. Output Resistance

Dual Input Balanced Output Differential Amplifier


## AC Analysis of Differential Amplifier

Important consideration for AC Analysis

1. Both $Q_{1} \& Q_{2}$ are identical BJTs
2. Resistances $R_{s 1}=R_{s 2}=R_{s} \& R_{C 1}=R_{C 2}=R_{C}$
3. DC supplies will be grounded
4. BJTs are replace with equivalent r-model


## AC Analysis - Voltage Gain

Differential Voltage Gain $A_{d}=\frac{V_{o}}{V_{1}-V_{2}}$

$$
V_{o}=i_{c 1} R_{c}-i_{c 2} R_{c}
$$

KVL in loop I: $\quad V_{1}=R_{S} i_{b 1}+r_{e} i_{e 1}+\left(i_{e 1}+i_{e 2}\right) R_{E}$
KVL in loop II: $\quad V_{2}=R_{S} i_{b 2}+r_{e} i_{e 2}+\left(i_{e 1}+i_{e 2}\right) R_{E}$
Substitute $i_{b 1}=\frac{i_{e 1}}{\beta}$ and $i_{b 2}=\frac{i_{e 2}}{\beta}$

$$
\begin{aligned}
V_{1} & =R_{S} \frac{i_{e 1}}{\beta}+r_{e} i_{e 1}+\left(i_{e 1}+i_{e 2}\right) R_{E} \\
V_{1} & =\left(\frac{R_{S}}{\beta}+r_{e}+R_{E}\right) i_{e 1}+R_{E} i_{e 2}
\end{aligned}
$$



Similarly, $\quad V_{2}=R_{E} i_{e 1}+\left(\frac{R_{s}}{\beta}+r_{e}+R_{E}\right) i_{e 2}$

## AC Analysis - Voltage Gain

Consider typical values $R_{S} / \beta=50 / 100$. Let's ignore $R_{S} / \beta$ as $R_{S} / \beta \ll R_{E} \& r_{e}$

$$
\begin{aligned}
& \left(r_{e}+R_{E}\right) i_{e 1}+R_{E} i_{e 2}=V_{1} \\
& R_{E} i_{e 1}+\left(r_{e}+R_{E}\right) i_{e 2}=V_{2}
\end{aligned}
$$

Solve using Crammer's Rule

$$
\left[\begin{array}{cc}
r_{e}+R_{E} & R_{E} \\
R_{E} & r_{e}+R_{E}
\end{array}\right]\left[\begin{array}{c}
i_{e 1} \\
i_{e 2}
\end{array}\right]=\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right]
$$

Determinant of resistance matrix

$$
\Delta=\left|\begin{array}{cc}
r_{e}+R_{E} & R_{E} \\
R_{E} & r_{e}+R_{E}
\end{array}\right|=\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}
$$

$$
\Delta_{1}=\left|\begin{array}{cc}
V_{1} & R_{E} \\
V_{2} & r_{e}+R_{E}
\end{array}\right|=\left(R_{E}+r_{e}\right) V_{1}-V_{2} R_{E}
$$

Current $i_{e 1}$ is given as

$$
i_{e 1}=\frac{\Delta_{1}}{\Delta}=\frac{\left(R_{E}+r_{e}\right) V_{1}-V_{2} R_{E}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}=i_{c 1}
$$

$$
\Delta_{2}=\left|\begin{array}{cc}
r_{e}+R_{E} & V_{1} \\
R_{E} & V_{2}
\end{array}\right|=\left(R_{E}+r_{e}\right) V_{2}-V_{1} R_{E}
$$

Current $i_{e 2}$ is given as

$$
i_{e 2}=\frac{\Delta_{2}}{\Delta}=\frac{\left(R_{E}+r_{e}\right) V_{2}-V_{1} R_{E}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}=i_{c 2}
$$

## AC Analysis - Voltage Gain

Output voltage $V_{o}=R_{c}\left(i_{c 1}-i_{c 2}\right)$
Put values of $i_{c 1}$ and $i_{c 2}$

$$
\begin{gathered}
V_{o}=R_{C}\left(\frac{\left(R_{E}+r_{e}\right) V_{1}-R_{E} V_{2}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}-\frac{\left(R_{E}+r_{e}\right) V_{2}-R_{E} V_{1}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}\right) \\
V_{o}=R_{c}\left(\frac{R_{E} V_{1}+r_{e} V_{1}-R_{E} V_{2}-R_{E} V_{2}-r_{e} V_{2}+R_{E} V_{1}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}\right) \\
V_{o}=R_{c}\left(\frac{2 R_{E} V_{1}+r_{e} V_{1}-2 R_{E} V_{2}-r_{e} V_{2}}{r_{e}^{2}+2 R_{E} r_{e}+R_{E}^{2}-R_{E}^{2}}\right)
\end{gathered}
$$

$$
\begin{gathered}
V_{o}=R_{c}\left(\frac{\left(2 R_{E}+r_{e}\right) V_{1}-\left(2 R_{E}+r_{e}\right) V_{2}}{r_{e}^{2}+2 R_{E} r_{e}}\right) \\
V_{o}=R_{c}\left(\frac{\left(2 R_{E}+r_{e}\right)\left(V_{1}-V_{2}\right)}{r_{e}\left(r_{e}+2 R_{E}\right)}\right) \\
V_{o}=\frac{R_{C}}{r_{e}}\left(V_{1}-V_{2}\right)
\end{gathered}
$$

Finally, Voltage Gain the Differential Amplifier is given as

$$
A_{d}=\frac{V_{o}}{V_{1}-V_{2}}=\frac{R_{C}}{r_{e}}
$$

## AC Analysis - Input Resistance

Input Resistance at first source of a Dual Input Balanced Output Differential Amplifier are given as

$$
R_{i 1}=\left.\frac{V_{1}}{i_{b 1}}\right|_{V_{2}=0}=\left.\frac{V_{1}}{i_{e 1} / \beta}\right|_{V_{2}=0}=\left.\frac{\beta V_{1}}{i_{e 1}}\right|_{V_{2}=0}
$$

Putting the value of $i_{e 1}$

$$
\begin{gathered}
R_{i 1}=\frac{\beta V_{1}}{\left.\frac{\left(R_{E}+r_{e}\right) V_{1}-V_{2} R_{E}}{\left(r_{e}+R_{E}\right)^{2}-R_{E}^{2}}\right|_{V_{2}=0}} \\
R_{i 1}=\frac{\beta r_{e} V_{1}\left(r_{e}^{2}+2 R_{E} r_{e}+R_{E}^{2}-R_{E}^{2}\right)}{\left(R_{E}+r_{e}\right) V_{1}} \\
R_{i 1}=\frac{\beta r_{e}\left(r_{e}+2 R_{E}\right)}{R_{E}+r_{e}}
\end{gathered}
$$

Since $\left(R_{E} \approx 3 k\right) \gg\left(r_{e} \approx 30\right)$ therefore, $r_{e}+2 R_{E} \approx 2 R_{E}$ and $r_{e}+R_{E} \approx R_{E}$.

Finally, input resistance is given as

$$
R_{i 1}=2 \beta r_{e}
$$

Similarly, Input Resistance at second input terminal is given as

$$
R_{i 2}=\left.\frac{V_{2}}{i_{b 2}}\right|_{V_{1}=0}=\frac{\beta r_{e}\left(r_{e}+2 R_{E}\right)}{R_{E}+r_{e}}=2 \beta r_{e}
$$

## AC Analysis - Output Resistance \& Current Gain

Output Resistance is measured at the collector terminal with respect to ground

$$
R_{o 1}=R_{o 2}=R_{C}
$$

Current Gain of a Differential Amplifier is undefined.
Differential Amplifier is generally used as Voltage Amplifier and not as Current or Power Amplifier.


## Numerical Problem

A Differential Amplifier with has $R_{C}=2.2 k \Omega$, $R_{E}=4.7 k \Omega, \quad R_{S 1}=R_{S 2}=50 \Omega, \quad V_{C C}=+10 \mathrm{~V}$, $V_{E E}=-10 \mathrm{~V}, \beta=100$ and $V_{B E}=0.7$. Determine its (a) Voltage Gain (b) Input Resistance (c) Output Resistance

Differential Voltage Gain formula is

$$
A_{d}=\frac{V_{o}}{V_{d}}=\frac{R_{C}}{r_{e}}
$$

However, $r_{e}=\frac{25 \mathrm{mV}}{I_{E}}$ at room temperature. But $I_{E}=$ ?


## Numerical Problem

During DC Analysis we following expression to determine $I_{E}$

$$
I_{C}=\frac{V_{E E}-V_{B E}}{\left(R_{S} / \beta+2 R_{E}\right)}=I_{E}
$$

Putting given values

$$
I_{E}=\frac{10-0.7}{(50 / 100+2 * 4.7 k)}=\mathbf{0 . 9 8 9 m} \mathbf{A}
$$

Further,

$$
r_{e}=\frac{25 m V}{I_{E}}=\frac{25 m V}{0.989 m A}=25.3 \Omega
$$



## Numerical Problem

Using Voltage Gain formula

$$
A_{d}=\frac{R_{C}}{r_{e}}=\frac{2.2 k \Omega}{25.3 \Omega}=\mathbf{8 6 . 9 6}
$$

Input Resistances are given as

$$
R_{i 1}=R_{i 2}=2 \beta r_{e}=2 * 100 * 25.3 \Omega=5.06 \mathbf{k} \Omega
$$

Output Resistances are given by

$$
R_{o 1}=R_{o 2}=R_{C}=2.2 \boldsymbol{k} \Omega
$$



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## Thank You

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