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LINEAR INTEGRATED CIRCUITS

3-08

Butterworth LPF, HPF & Higher Order Filters

Outline

1. Active Filters
2. Butterworth Active Filters
3. First Order Low Pass Filter
4. Second Order Low Pass Filter
5. First Order High Pass Filter
6. Second Order High Pass Filter
7. Higher Order Filters

Filter Classification

Analog & Digital Filters

Digital filters use digital techniques to filter analog signals.

Analog filters are meant to filter analog signals. These can be, further, classified as Active and Passive Filters.

Passive & Active Filters

Passive filters use linear components, i.e., Resistors, Capacitors and Inductors.

Filters that uses Op-Amp & Transistors, etc. semiconductor components in addition to Resistors & Capacitor. Inductors are not used in filters due to their large size, cost and magnetic interference.

Advantages of Active Filters

Gain & Frequency Adjustment

In active filters, Op-Amp provides voltage gain in pass band rather than attenuation like passive filters.

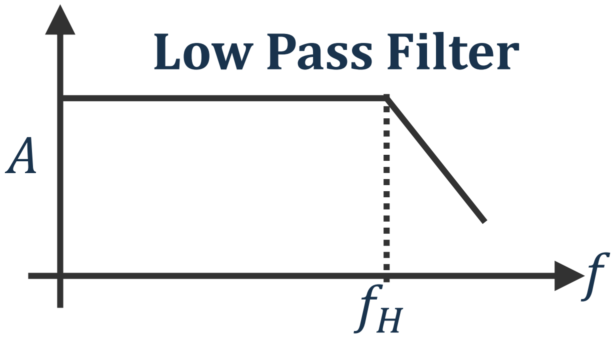
No Loading Problem

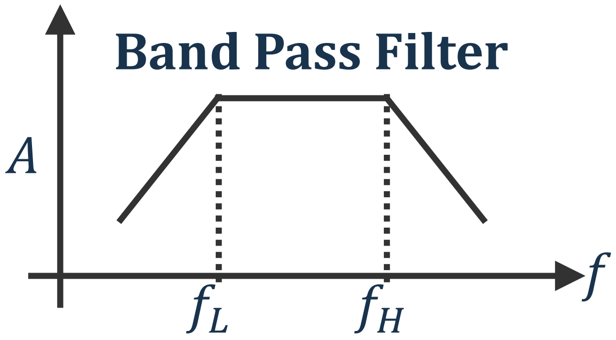
Op-Amp offers high input impedance and low output impedance. Therefore, active filter do not pose loading problem of the source and load.

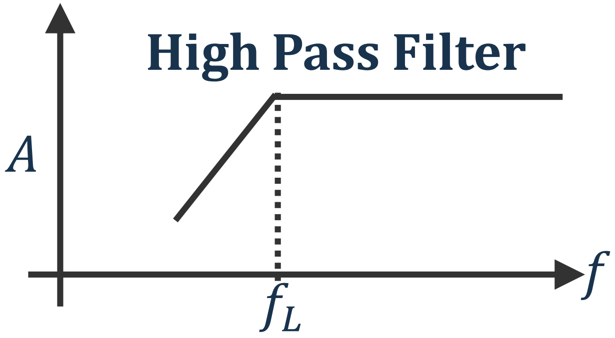
Cost

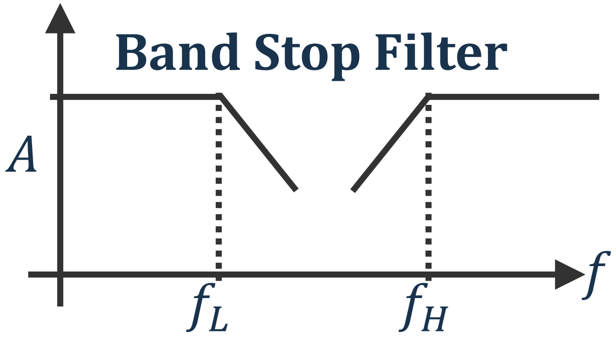
Active filters do not use inductors, therefore, are cost effective than passive filters.

Frequency Response of Filters









First Order Butterworth LPF

The output of non-inverting amplifier is

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

Where v_1 is voltage at the non-inverting input terminal.

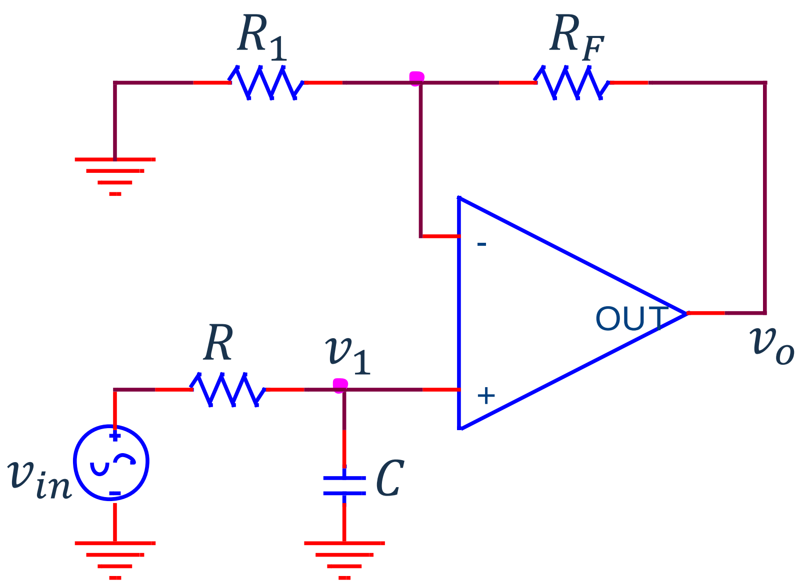
This v_1 voltage is given as

$$v_1 = \frac{-jX_C}{R - jX_C} v_{in}$$

Put $-jX_C = \frac{1}{j2\pi fC}$ in above equations, and simplify

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{v_{in}}{1 + j2\pi fRC}$$

$$\frac{v_o}{v_{in}} = \frac{A_F}{1 + j2\pi fRC} = \frac{A_F}{1 + j\left(\frac{f}{f_H}\right)}$$

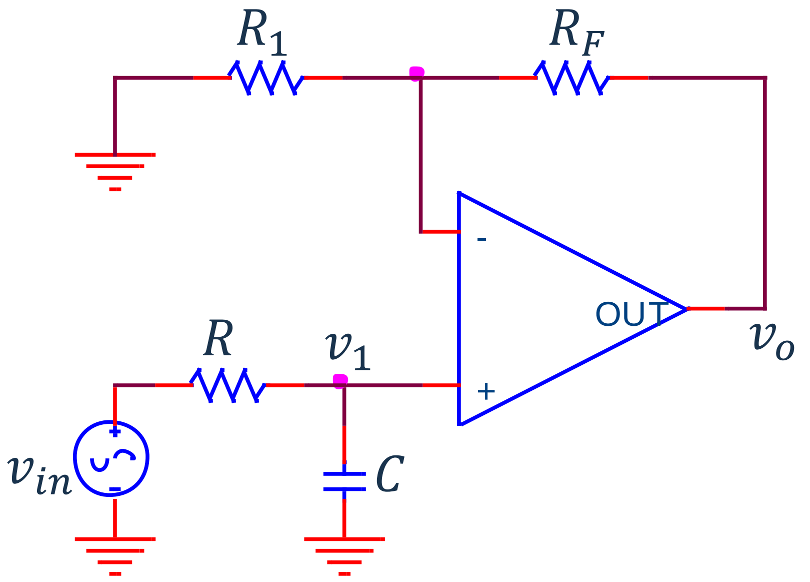


Here $A_F = 1 + \frac{R_F}{R_1}$, f is the input signal frequency, $f_H = \frac{1}{2\pi RC}$ high cut-off frequency

First Order Butterworth LPF

Magnitude and phase angle can be determined as

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+(f/f_H)^2}} \qquad \text{and} \qquad \phi = -\tan^{-1}\left(\frac{f}{f_H}\right)$$



At very low frequencies, $f = 0\text{Hz}$

$$\left|\frac{v_o}{v_{in}}\right| \cong A_F$$

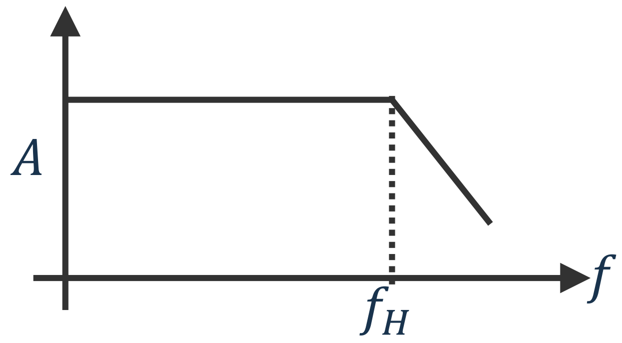
At frequency $f = f_H$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707\ A_F$$

$$\left|\frac{v_o}{v_{in}}\right| = A_F\ 20\log 0.707 = 3\text{dB}$$

For frequencies $f > f_H$

$$\left|\frac{v_o}{v_{in}}\right| < A_F$$

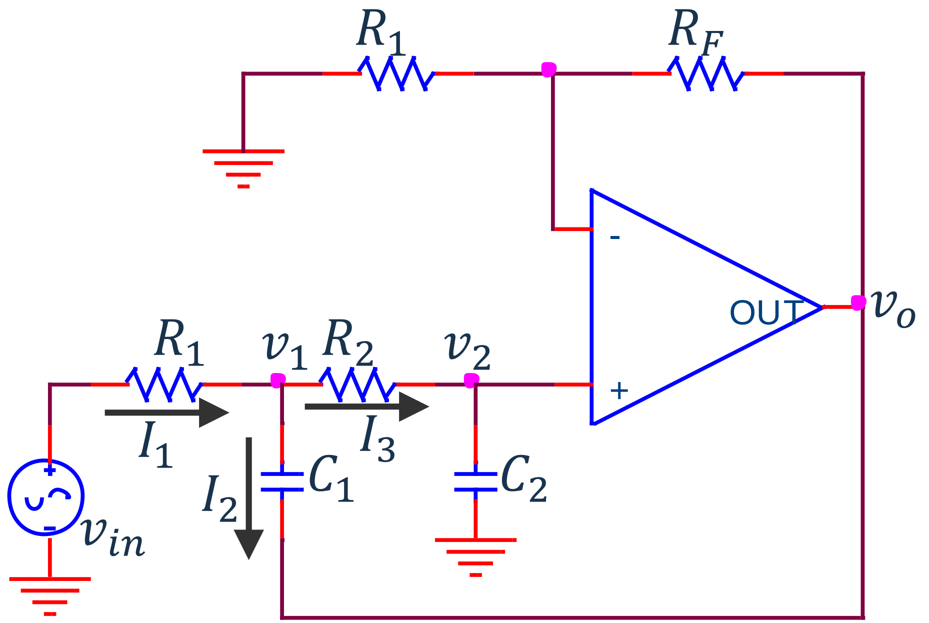


Second Order Butterworth LPF

Applying KCL at node v_1

$$I_1 = I_2 + I_3$$

$$\frac{v_{in} - v_1}{R_1} = \frac{v_1 - v_o}{1/sC_1} + \frac{v_1 - v_2}{R_2} \qquad (1)$$



Using voltage divider rule

$$v_2 = \frac{1/sC_3}{R_3 + 1/sC_3} v_1 = \frac{v_1}{1 + sC_3R_3}$$

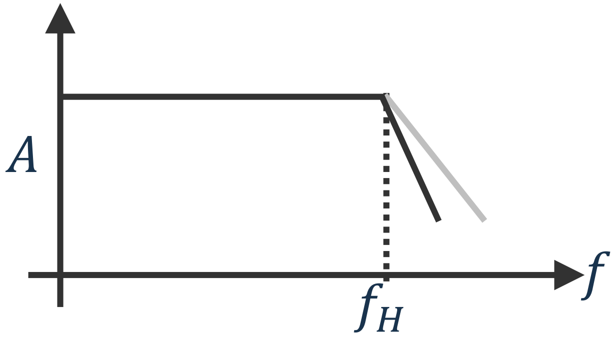
$$v_1 = (1 + sC_3R_3)v_2$$

Put in (1) and simplify

$$v_2 = \frac{R_3 v_{in} + sR_2R_3C_2v_o}{(sR_3C_3 + 1)(R_2 + R_3 + sR_2R_3C_2) - R_2}$$

Finally the output of non-inverting amplifier

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_2 = A_F \frac{R_3 v_{in} + sR_2R_3C_2v_o}{(sR_3C_3 + 1)(R_2 + R_3 + sR_2R_3C_2) - R_2}$$



Second Order Butterworth LPF

Further, simply to get

$$\frac{v_o}{v_{in}} = \frac{A_F}{s^2 + \frac{s(R_3C_3 + R_2C_3 + R_2C_2 - A_F R_2C_2)}{R_2R_3C_2C_3} + \frac{1}{R_2R_3C_2C_3}}$$

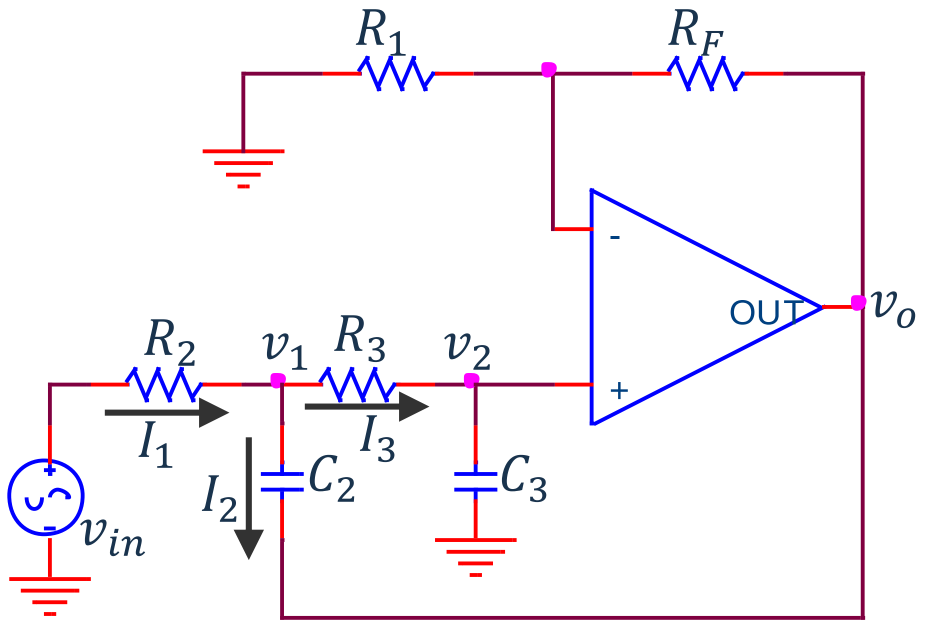
For real roots of the quadratic equation in denominator, put

$$\omega_H^2 = \frac{1}{R_2R_3C_2C_3}$$

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

For simplicity, assume $R_2 = R_3 = R$ and $C_2 = C_3 = C$

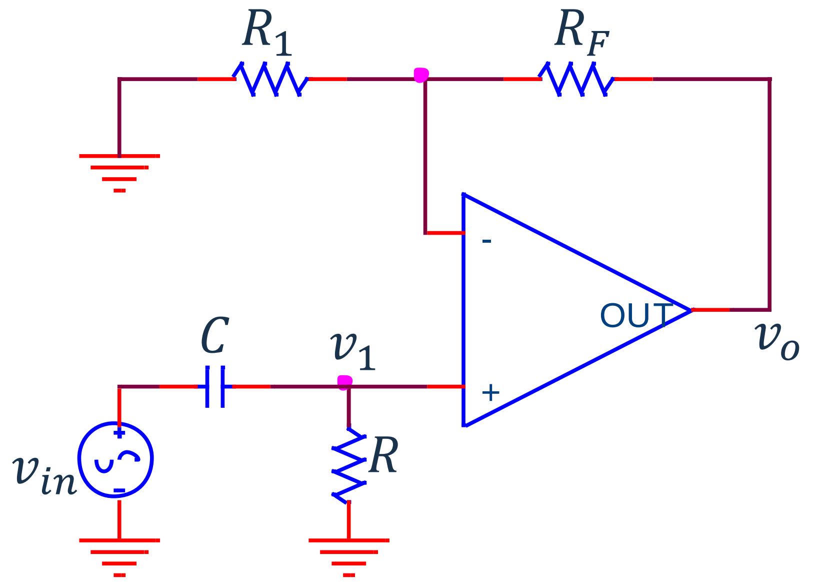
$$f_H = \frac{1}{2\pi RC}$$



First Order Butterworth HPF

The output of non-inverting amplifier is

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$



Where v_1 voltage is given as

$$v_1 = \frac{R}{R + \frac{1}{j2\pi fC}} v_{in} = \frac{j2\pi fCR}{1 + j2\pi fCR} v_{in}$$

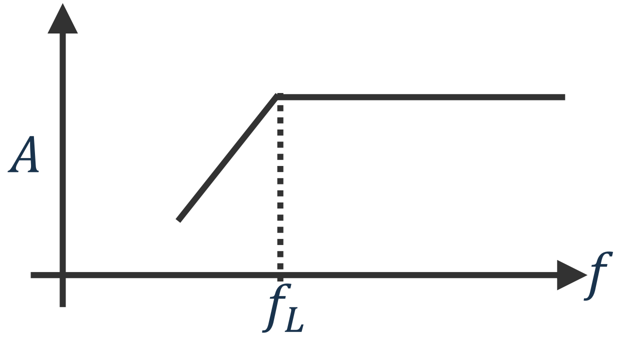
Output of the non-inverting amplifier

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{j2\pi fRC}{1 + j2\pi fRC} v_{in}$$

$$\frac{v_o}{v_{in}} = A_F \frac{j2\pi fRC}{1 + j2\pi fRC} = A_F \frac{j(f/f_L)}{1 + j(f/f_L)}$$

Here, $f_L = \frac{1}{2\pi RC}$ low cut-off frequency. Magnitude can be determined as

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$



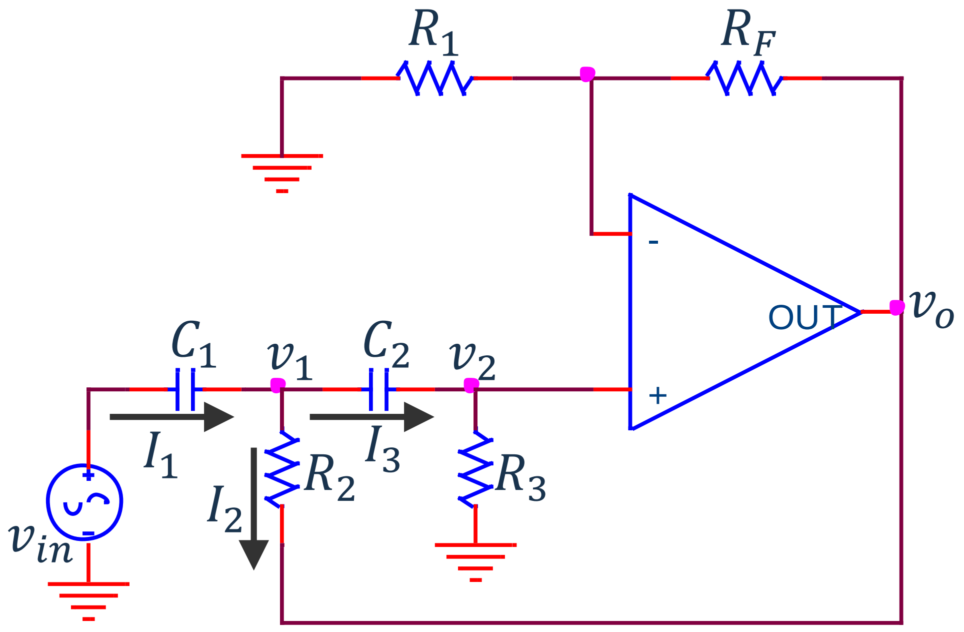
Second Order Butterworth LPF

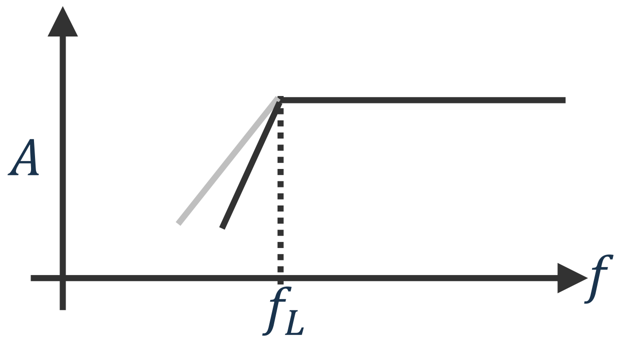
As we determined the cut-off frequency in Second Order Butterworth LPF, we can determine cut-off frequency of Second Order Butterworth HPF also.

$$f_L = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

For simplicity, assume $R_2 = R_3 = R$ and $C_2 = C_3 = C$

$$f_L = \frac{1}{2\pi RC}$$





Higher Order Filters

In first order filters, the gain of the filters changes at rate 20dB/decade.

Whereas, in second order the gain changes at the rate of 40dB/decade.

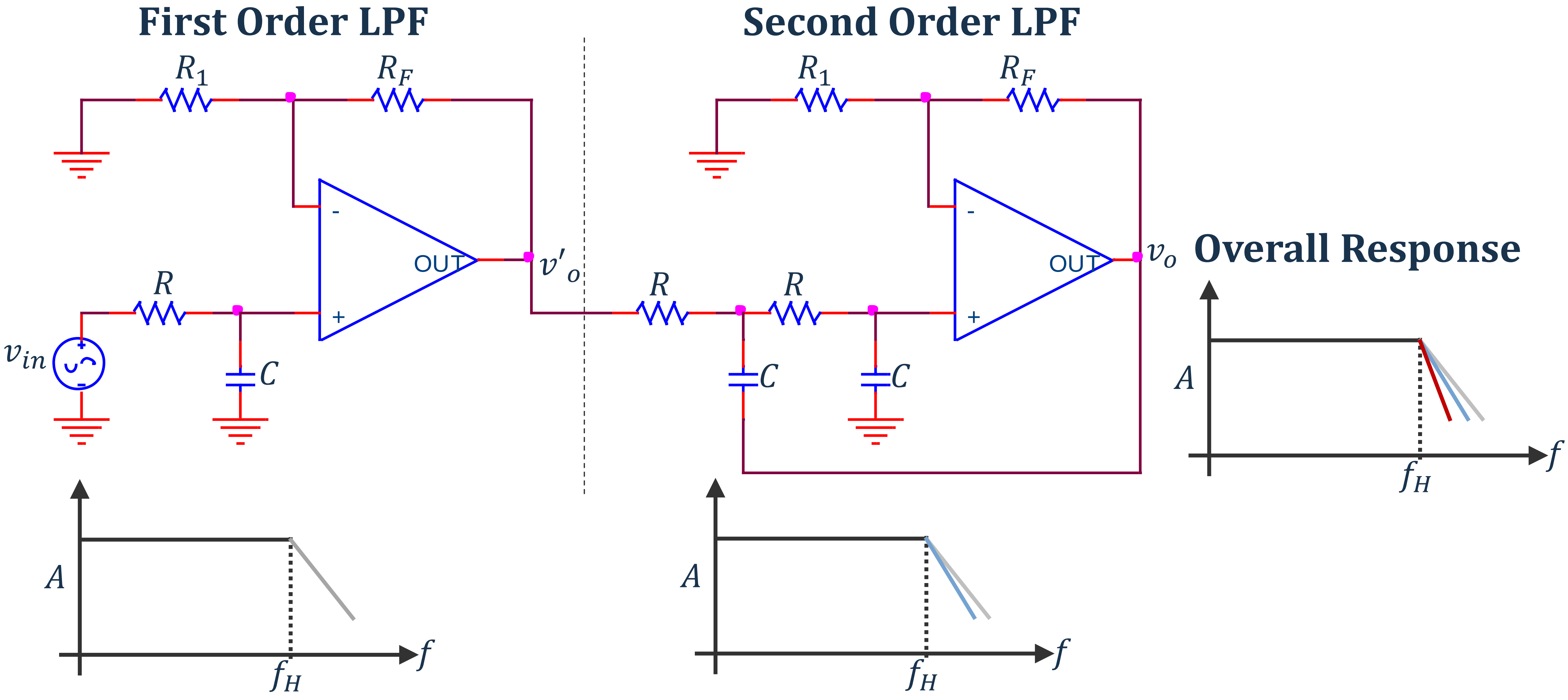
This means that as the order of filter is increased the stop band response of the filter approaches ideal filter response.

Higher order filter can be design by cascading first and second order filters.

For example, third order filter will require one first order and one second order filter.

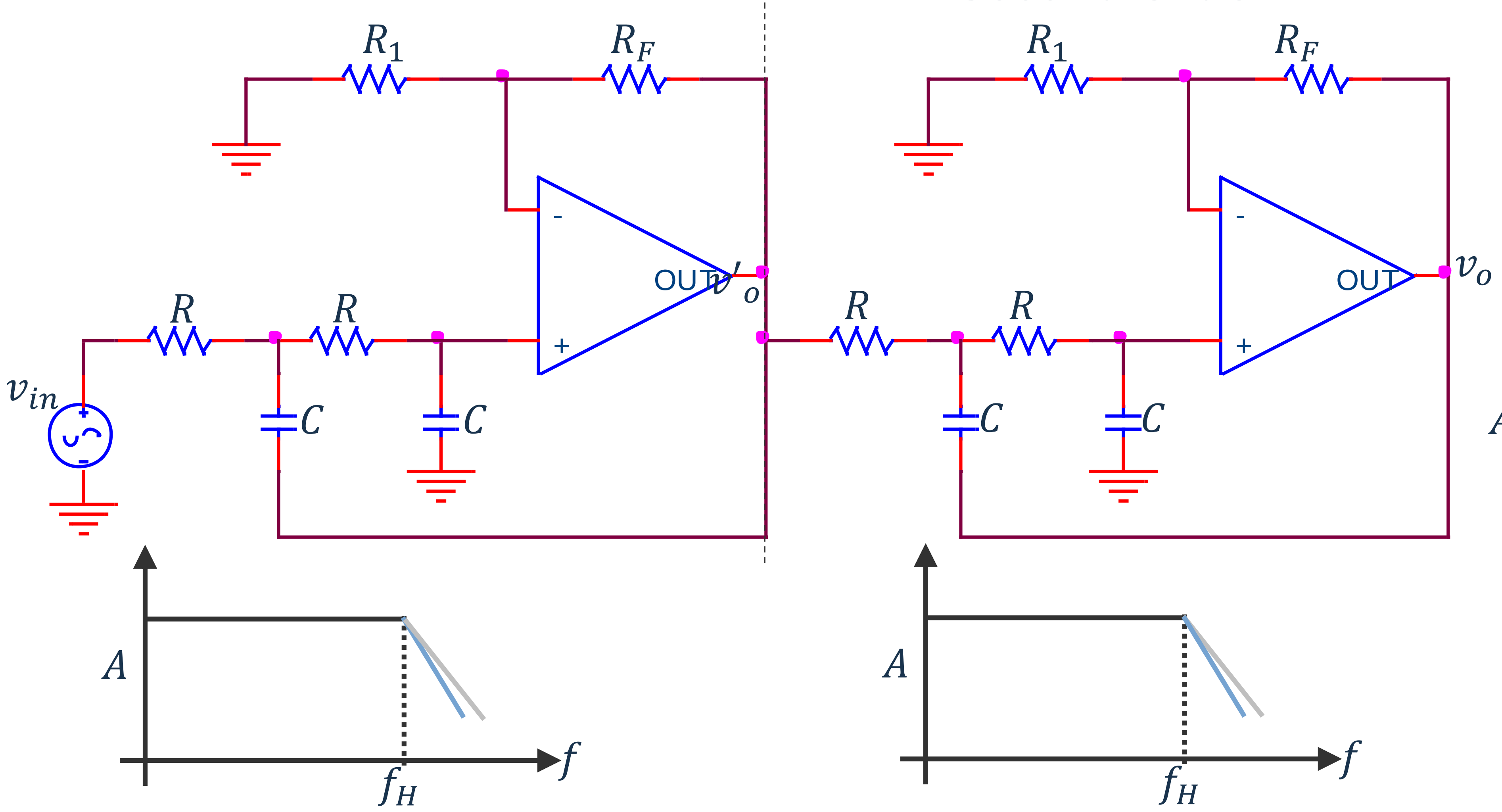
Whereas, fourth order filter will require two second order filter connected in series.

Third Order LPF

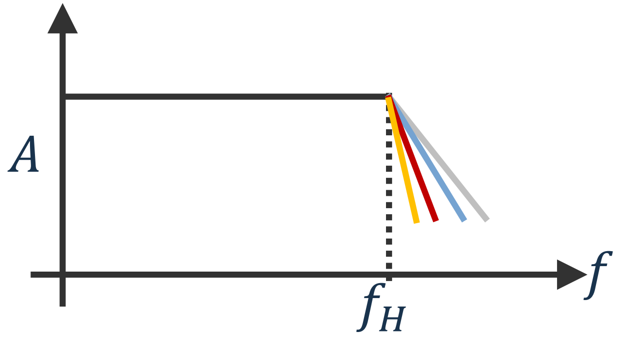


Fourth Order LPF

Second Order LPF **Second Order LPF**



Overall Response



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Thank You