The Firefly Optimization Algorithm: Convergence Analysis and Parameter Selection

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ABSTRACT
The bio-inspired optimization techniques have obtained great attention in recent years due to its robustness, simplicity and efficiency to solve complex optimization problems. The firefly Optimization (FA or FFA) algorithm is an optimization method with these features. The algorithm is inspired by the flashing behavior of fireflies. In the algorithm, randomly generated solutions will be considered as fireflies, and brightness is assigned depending on their performance on the objective function. The algorithm is analyzed on basis of performance and success rate using five standard benchmark functions by which guidelines of parameter selection are derived. The tradeoff between exploration and exploitation is illustrated and discussed.

General Terms
Optimization, metaheuristic, firefly algorithm, analysis, convergence, parameter selection, performance.

Keywords
Optimization; firefly algorithm; convergence; parameter selection;

1. INTRODUCTION
The basic meaning of optimization in our daily lives is to do better in a field and in area of computational intelligence, optimization can be basically described as finding a parameter in a function which can make the solution better among all the possible solutions and the best value is known as optimum solution. [1]. In order to solve optimization problems, optimization algorithms are used which can be classified in a simple way into deterministic algorithms and stochastic algorithms. Deterministic algorithms are those whose behavior can be completely predicted as they are presented with same set of input and algorithm does same computations and produce same set results every time [2], [3].

Generally stochastic algorithms are divided into heuristic and metaheuristic algorithms. Heuristic algorithms find solutions in a reasonable amount of time but there is no guarantee that optimal solutions are reached [4], [5]. On the other hand metaheuristic uses certain tradeoff a randomization and local search, as randomization provides a good way to move away from local search to the search on global scale which means metaheuristic algorithms intend to be suitable for global optimization. Most stochastic algorithms can be considered as metaheuristic and good examples are Genetic Algorithm (GA) [6], [7]. Many modern metaheuristic algorithms were developed based on swarm intelligence in nature like PSO and AFSA. Firefly algorithm is a metaheuristic algorithm developed by Dr. Xin-Shi Yang[3]. This algorithm is based on the natural behavior of fireflies which is based on the phenon--menon of bioluminescence. Fireflies in nature are capable of producing light thanks to special photogenic organs situated very close to the body surface behind a window of translucent cuticle [3]. This light helps them to communicate with each other and attract prey or other fireflies.

2. Basic Concepts
2.1 Optimization Problem
An optimization problem has basically three components: a function to optimize, possible solution set to choose a value for the variable from, and the optimization rule, which will be either maximized or minimized[8], [9]. Since one can switch between minimization and maximization problems by multiplying the objective function by negative one, analyzing either minimization or maximization problem is enough[10]. A typical maximization problem can be given as follows:

$$\text{Max } f(x)$$
$$\text{s.t. } x \in S \subseteq R^n \tag{1}$$

A solution for this problem is a member of set S which gives the maximum value of f(x)compared to all elements in S. Mathematically, x* is a solution if and only if $x^* \in S$ and $f(x^*) \geq f(x)$, for all $x \in S$. The aim of all solution methods and algorithms is to find such x*[11], [12], [13].

2.2 Firefly algorithm
2.2.1 Behavior of flies
The bioluminescence processes is responsible for flashing light of fireflies. There are many theories regarding reason and importance of flashing light in firefly's life cycle but many of them converge to mating phase [14]. The basic objective of flashing light is to attract mating partner The pattern of these rhythmic flashes is unique based upon rhythm of flashes, rate of flashing and amount of time for which flashes are observed [15]. This pattern attract both the males and females to each other and female of a species respond to individual pattern of male of same species . According to the inverse square law, intensity of the light I, keeps on decreasing as the distance r increases in terms of I α 1/r2. Air also acts as absorbent and light gets weaker with increasing distance [16]. Combining these two factors reduce the visibility of fireflies to a limited distance normally few hundred meters at night which is sufficient for fireflies to communicate with each other.

2.2.2 Concept
Firefly algorithm is based upon idealizing the flashing characteristic of fireflies. The idealized three rules are:-

• All fireflies are considered as unisex and irrespective of the sex one firefly is attracted to other fireflies

•The Attractiveness is proportional to their brightness, which means for any two flashing fireflies, the movement of firefly is from less bright towards the brighter one and if no one is brighter than other it will move randomly. Furthermore they both decrease as their distance increases.

• The landscape of the objective function directly affects the brightness of the firefly [3] [17].

For a maximization problem, the brightness is proportional to the objective function's value. Other forms of the brightness could be defined in same way to the fitness function as in genetic algorithms.

2.2.3 Light intensity and attractiveness

Firefly algorithm is based on two important things, first is the variation in light intensity and second is formulation of attractiveness. For simplicity it is assumed that attractiveness of firefly is determined by its brightness which is connected with objective function [18].

At particular location x, the brightness I of a firefly can be chosen as $I(x) \propto f(x)$ for a maximization problem.

The attractiveness β is relative, which means it should be judged by other fireflies, so it will differ with distance r_{ij} between firefly i and firefly j. As already stated, the light intensity decreases with distance from its source and light is also absorbed by air, so attractiveness should be allowed to vary with varying degree of absorption[3], [17]. So in simplest form, light intensity I(r) varies according to inverse square law.

$$I(r) = \frac{I_s}{r^2} \quad (2)$$

I_s is the intensity at the source. The light intensity I varies with the distance r having a fixed light absorption coefficient γ i.e.

$$I = Ioe^{-\gamma r} \quad (3)$$

where Io is the initial light intensity, In order to avoid the singularity at r = 0 in the expression Is/r2 the combined effect of both the inverse square law and absorption can be approximated as the following Gaussian form[17], [19]:

$$I = Ioe^{-\gamma r^2} \quad (4)$$

Firefly's attractiveness β is proportional to the light intensity seen by adjacent fireflies which can be defined as:

$$\beta = \beta o e^{-\gamma r^2} \quad (5)$$

where β0 is the attractiveness at r = 0. Since it is often faster to calculate1/(1 + r2) than an exponential function, the above function, if necessary, can be approximated as shown in (6).

$$\beta = \frac{\beta o}{(1+\gamma r^2)} \quad (6)$$

Both (5) and (6) define a characteristic distance Γ 1/γ over which the attractiveness is changing significantly from β_o to βoe-1 for equation (5) or $\beta_o/2$ for equation (6)[3], [16]. In the real time implementation, β(r) is the attractiveness function which can be can any monotonically decreasing function like the following.

$$\beta(r) = \beta o e^{-\gamma r^m} (m \geq 1) \quad (7)$$

For a fixed, the characteristic length becomes

$$\Gamma = \gamma^{\frac{-1}{m}} \to 1, m \to \infty \quad (8)$$

Conversely, γ can be used as typical initial value for a specific length scale Γ in an optimization problem. That is

$$\gamma = \frac{1}{\Gamma^m} \quad (9)$$

The distance between any two fireflies is calculated using Cartesian distance method.

$$r_{i,j} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (10)$$

In (10) $x_{i,k}$ is the k^{th} component of spatial coordinate x_i of i^{th} firefly. In 2-D case, we have

$$r_{i,j} = \sqrt{(x_i - x_j)^2 - (y_i - y_j)^2} \quad (11)$$

Firefly i is attracted to brighter firefly j and its movement is determined by

$$x_i = x_i + \beta o e^{-\gamma r_{i,j}^2} (x_j - x_i) + \alpha € _i \quad (12)$$

Second component is used for the attraction and third component is used for randomization with α being the randomization parameter, and €i is a vector of random numbers being drawn from a Gaussian distribution or uniform distribution[3], [17]. For example, the simplest form is €i could be replaced by (rand − ½) where rand is a random number generator distributed in a uniform range of [0, 1]. It should be noted that (12) is a random walk partial towards the brighter fireflies like if βo = 0, it becomes a simple random walk. The most important parameter in firefly algorithm is γ, it plays very crucial role in determining the speed of convergence and how FA algorithm behaves. Theoretically, γ Є [0, ∞) but in actual practice, γ O(1) is determined by the characteristic length Γ of the system to be optimized [20], [21]. So for almost all the applications it varies from 0.1 to 10. The firefly algorithm is given below:

Firefly algorithm( )
1. Objective function f(x), x=(x1,x2,…,xd)^T
2. Initialize a population of fireflies xi(i= 1,2, . . . , n)
3. Define light absorption coefficient γ
4. While (t<MaximumGenerations)
5. For i=1:n (all n fireflies)
6. For j=1:i
7. Light intensity Ii at xi is determined by f(xi)
8. If (Ii > Ij)
9. Move firefly i towards j in all d dimensions
10. Else
11. Move firefly i randomly
12. End If
13. Attractiveness changes with distance r via exp[-γr2]
14. Determine new solutions and revise light intensity
15. End for j
16. End for i
17. Rank the fireflies according to light intensity and find the current best
18. End while

3. PARAMETER SELECTION GUIDELINES

3.1 Effect of random numbers

The rigorous analysis of optimization algorithm with random numbers is beyond the scope of this paper. By including random numbers, state space exploration is improved but it slows down the convergence i.e. it prevents premature convergence to non-optimal points.

3.2 Parameter tuning heuristic

In addition to parameters to α and γ and to effect of random numbers discussed above, the convergence of algorithm is indirectly proportional to number of fireflies i.e. more the number of fireflies, more iterations are needed to converge [22]. The best rate of convergence i.e. best tradeoff between exploration and exploitation strongly depends upon many things like number of local optima, function being optimized, distance from global solution, position of global solution in search domain, size of domain etc. It is nearly impossible to find set of unique parameters for algorithm which would work in all cases but the following empirical procedure, based on the above considerations, was found to work in practice [19], [21].

3.3 Convergence

The algorithm is run several times until convergence with convergent parameter set like mentioned in "Optimization experiments" section. If different results are obtained in most runs, the convergence rate is too high; the algorithm converges prematurely to non-optimal points. Slow convergent parameter set should be used i.e. larger number of fireflies, greater domain size [22]. If same result is obtained again and again, but during a large fraction of the algorithm

Name	Formula	Range	Goal for f
Sphere	$\boldsymbol{f_0(\vec{x}) = \sum_{i=1}^{n} x_i^2}$	$[-100,100]^n$	0.01
Ackley	$\boldsymbol{F(\vec{x}) = -20.exp\left(-0.2\sqrt{\frac{i}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi.x_i)\right) + 20 + e}$	$[-30,30]^n$	1
Rosenbrock	$f_1(\vec{x}) = \sum_{i=1}^{n-1}(100(x_{i+1} - x_j^2)^2 + (x_i - 1)^2$	$[-30,30]^n$	100
Rastrigin	$f_2(\vec{x}) = \sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10)$	$[-5.12,5.12]^n$	100
Griewank	$f_3(\vec{x}) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600,600]^n$	0.1

Table 1: Optimization functions

iterations no better points are found, the convergence rate is too low, fireflies do not focus the search around promising candidate solutions quickly enough. A more quickly convergent parameter set should be selected, smaller number of fireflies, smaller dimension. If consistent results are obtained, convergence rate is good and the same parameter set, domain size should be used to solve similar problems.

4. OPTIMIZATION EXPERIMENTS

4.1 Test conditions

The firefly optimization algorithm was used for the optimization of four benchmark functions. Three sets of parameters were used. Parameter set 1 (α=0.1, γ = 0.01), set 2 (α=0.25, γ = 0.1) and set 3(α=0.5, γ = 1) were selected by the author in the algorithm convergence domain after a large number of simulation experiments. The functions, the number of dimensions (n), the admissible range of the variable (x), the optimum and the goal values are summarized in Table 1. The number of iterations required to reach the goal was recorded. Each optimization experiment was run 25 times with random initial values of x and v in range [x_{min}, $x_{max.}$] indicated in table 1. During the optimization process the fireflies were not allowed to fly outside the region defined by [x_{min}, x_{max}].

4.2 Optimization results and discussion

4.2.1 Effect of number of fireflies in domain(N)

In most cases, increasing the number of fireflies increased the number of required algorithm iterations as indicated by average, median, minimum and maximum values reported in Table 2. The success rate of algorithm (fraction of number of runs it reached the goal) was constant in all cases except for Rastrigin function. Since in real life applications the optimization cost is usually dominated by evaluations of objective function, the expected number of function evaluations was retained as main algorithm performance criterion. It takes into account success rate, number of fireflies and number of algorithm iterations. Best results are obtained with minimum number of fireflies. Usually few numbers of fireflies gave good success rate except in case of Rastrigin function.

Parameter set 1 (α=0.1 and γ = 0.01), set 2(α=0.25 and γ = 0.1) and set 3(α=0.5 and γ = 1)																
		Number of algorithm iterations to achieve the goal												Expected number of function evaluations[b]		
		Average			Median			Minimum			Maximum					
Function	N	Set1	Set2	Set3	Set1	Set2	Set3	Set1	Set2	Set3	Set1	Set2	Set3	Set1	Set2	Set3
Sphere	15	6355	7395	8104	6377	7388	8098	6239	7308	7893	6440	7479	8312	95325	110925	121560
	30	6303	7331	8104	6291	7348	8104	6202	7173	8001	6452	7446	8225	189090	219930	243120
	60	6256	7176	7994	6213	7167	7986	6149	7036	7851	6398	7303	8123	375360	430560	479640
Ackley	15	1926	2769	3758	1901	2791	3778	1720	2628	3557	2176	2883	3887	28890	41535	56370
	30	1709	2693	3649	1696	2659	3645	1568	2556	3574	1805	2801	3633	51270	80790	109470
	60	1862	2801	3709	1557	2669	3601	1402	2606	3494	1862	2801	3709	111720	168060	222540
Rosen-brock	15	2759	3655	4531	2771	3654	4520	2497	3563	4443	2938	3655	4531	41385	54825	67965
	30	2653	3541	4434	2617	3515	4435	2531	3386	4374	2653	3541	4434	79590	106230	133020
	60	2575	3470	4432	2597	3479	4438	2471	3316	4351	2574	3470	4432	154500	208200	265920
Rastrigin	15	1300	2561	3228	1259	2522	3225	1081	2275	2928	1510	3148	3712	19500	38415	48420
	30	1262	2351	3302	1174	2262	3197	909	1982	2974	1910	2709	3883	37860	70530	132080
	60	958	2340	3179	955	2164	3067	99	2017	2826	1272	2796	4060	57480	175500	211933
Griewank	15	5420	6512	7438	5408	6485	7483	5138	6365	7201	5605	6705	7562	81300	97680	111570
	30	5428	6356	7413	5448	6323	7413	5201	6252	7204	5606	6469	7514	162840	190680	222390
	60	5295	6293	7319	5279	6300	7310	5176	6156	7157	5371	6465	7449	317700	377580	439140

[a] Fraction of the number of optimization experiments in which the goal was reached

[b] (Number of particles in the swarm) × (Average number of iterations)/(Success rate)

Table 2: Firefly algorithm performance

4.2.2 Effect of objective function

Sphere and rosenbrock have single minimum while other functions have multiple local minima. Rastrigin and griewank have large scale curvature which guides the search towards global minimum. Only in case of rastrigin function, some time there was failure to achieve goal due to premature convergence to local minimum.

4.2.3 Effect of parameter γ and α

Parameter set 1 has the highest convergence rate than set 2 and set 3. Exploitation is favored compared to exploration of state space. The number of algorithm iterations for set 1 was generally smaller. Smallest number of function evaluations was achieved for parameter set 1. Convergence parameter is not the only important factor, convergence trajectory is also important because equal convergence rate doesn't yield good results. For n=15 with α=0.1 and γ = 0.01, the author obtained good results as shown in Table 2.

5. Summary

An analysis is made on Firefly algorithm considering its dynamic behavior and convergence as important aspects. Algorithm is analyzed using tools from discrete-time dynamic system and this analysis provides qualitative guidelines for general (random) algorithm parameter selection. Simulation experiments are conducted with three parameter sets, three parameter set of fireflies in domain and five benchmark functions. The speed of convergence-robustness tradeoff was discussed. Better results are achieved in small number of fireflies in domain and smaller value of α and γ. Further research is needed to clarify effect of randomness and their effect on convergence. Better parameter sets probably await discovery in the outlined algorithm convergence domain.

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